BSCCS2001: Graded Solutions
Week 5

1. Consider relation **R**(P, Q, C, A, B) having the following functional dependencies:
$\mathcal{F} = \{P \rightarrow QC, CA \rightarrow B, Q \rightarrow A, B \rightarrow P\}$
Then, which of the following is correct?

[MCQ: 2 points]

- ◯ P, B & Q are non-prime attributes.
- ◯ Only P and B are prime attribute.
- ◯ C, A & Q are non-prime attributes.
- √ P, B, Q, C & A are prime attributes.

Solution: Prime Attributes: The attributes that belong to any candidate key are called prime attributes.

Non-prime Attribute: The attributes that do not belong to any candidate key are called non-prime Attributes.

Find out the closure of individual attributes :
$P^+ = P$
$= PQC$ $\{P \rightarrow QC\}$
$= PQCA$ $\{Q \rightarrow A\}$
$= PQCAB$ $\{CA \rightarrow B\}$
$Q^+ = A$
$C^+ = C$
$A^+ = A$
$B^+ = P$
$= PQC$ $\{P \rightarrow QC\}$
$=PQCA$ $\{Q \rightarrow A\}$
$= PQCAB$ $\{CA \rightarrow B\}$

Here, P and B are candidate key, means they are prime attribute .
Now, let us check the combination of Q, C and A
$QC^+ = QC$
$= QCA$ $\{Q \rightarrow A\}$
$= QCAB$ $\{CA \rightarrow B\}$
$= QCABP$ $\{B \rightarrow P\}$

$CA^+ = CA$
$= CAB \ \{CA \rightarrow B\}$
$= CABP \ \{B \rightarrow P\}$
$= CABPQ \ \{P \rightarrow QC\}$
Here, CA and QC are also candidate key. Hence, P, B, Q C and A are prime attributes.

2. Consider relation **Z**$(P,\ Q,\ R,\ A,\ B,\ C)$ having functional dependencies

   $\mathcal{F} = \{P \to Q,\ P \to R,\ RA \to B,\ RA \to C,\ Q \to B\}$
   For $PA \to C$ to be the member of $\mathcal{F}^+$, which of the following is true?

   [MCQ: 2 points]

   - ◯ By augmenting $P \to R$ with A to get $PA \to RA$, and then reflexivity with $RA \to C$.
   - ◯ By reflexivity $P \to R$ with A to get $PA \to RA$, and then transitivity with $RA \to C$.
   - √ By augmenting $P \to R$ with A to get $PA \to RA$, and then transitivity with $RA \to C$.
   - ◯ By reflexivity $P \to R$ with A, to get $PA \to RA$, and then augmenting with $RA \to C$.

Solution:
Armstrong's Axioms:
if $\beta \subseteq \alpha$, then $\alpha \to \beta$ (reflexivity)
if $\alpha \to \beta$, then $\gamma\alpha \to \gamma\beta$ (augmentation)
if $\alpha \to \beta$, and $\beta \to \gamma$, then $\alpha \to \gamma$ (transitivity)

From the given set of FDs
$P \to R$, by augmentation with A, we get $PA \to RA$
also from FDs $RA \to C$, so by transitivity $PA \to C$. Hence, option 3 is correct.

3. The information of all students who have registered for the IIT Madras Online Degree course is given by the relation **studinfo**(*<u>studId</u>, name, state*). The relation **enroll** (*<u>studId</u>, courseId*) gives the list of courses for which each student has enrolled. Let **R** be the relation resulting from the natural join of **studinfo** and **enroll**.
   That is, **R** = **studinfo** ⋈ **enroll**.
   Then, which of the following statements is/are true? [MSQ:2 points]

   - √ The relations **studinfo** and **enroll** result from a lossless decomposition of relation *R*.
   - ◯ The relations **studinfo** and **enroll** result from a lossy decomposition of relation *R*.
   - √ The number of super keys of *R* is 8.
   - ◯ The number of super keys of *R* is 15.

Solution:
studinfo(*<u>studId</u>, name, state*)
enroll (*<u>studeId</u>,courseId*)

R = **studinfo** ⋈ **enroll** = **R**(*<u>studId</u>, name, state, courseId*)
Let us check, whether it is lossy or lossless decomposition:
studinfo ∪ **enroll** = *(studId, name, state, courseId)* = R
studinfo ∩ **enroll** = $studId \neq \phi$, here *studId* is primary key. So, we can determine **studinfo** ∩ **enroll** → **studinfo** or **studinfo** ∩ **enroll** → **enroll**.
Hence, The relation **studinfo** and **enroll** are lossless decomposition of relation **R**.

The number of super keys is given by 2^{n-1}
In relation **R**, the number of attributes is 4 i:e n = 4. So, the number of super keys is 8.

4. A relation **Z**$(P,\ Q,\ R,\ S,\ T,\ U,\ V)$ has the following set of functional dependencies: $\mathcal{F} = \{P \rightarrow Q, QR \rightarrow ST, PTU \rightarrow V\}$
What is the closure of the attribute set $\{P, R\}$ under $\mathcal{F}$?

[MCQ: 1 points]

- ◯ $P,\ R,\ S,\ T$
- ◯ $P,\ R,\ U,\ S$
- ◯ $P,\ Q,\ R,\ S,\ T,\ U$
- √ $P,\ R,\ Q,\ S,\ T$

Solution:
$(P, R)^+ = P,\ R$
$= P,\ R,\ Q\ \ (P \rightarrow Q)$
$= P,\ R,\ Q,\ S,\ T\ \ (QR \rightarrow ST)$
Thus, option 4 is correct.

5. Consider the relation **R**$(A,\ B,\ C,\ X,\ Y,\ Z)$ having the following functional dependencies $\mathcal{F} = \{AB \to C, C \to X, X \to Y, Y \to Z, Z \to B\}$
Which of the following is/are true? [MSQ: 3 points]

- √ AC and AY are candidate keys.
- √ All attributes are prime attributes.
- ◯ AC, XY and BC are candidate keys.
- ◯ XY and AZ are candidate keys.

Solution: From the given sets of functional dependencies, if we individually take the closure of $A,\ B,\ C,\ X,\ Y\ and\ Z$, it cannot determine the relation R. Thus, alone $A,\ B,\ C,\ X,\ Y\ and\ Z$ can't be a candidate key.
Since there is no incoming arrow to A, A will be the part of some candidate key, but by itself, it is not a candidate key.

Consider the closure of :
$AB^+ = AB$
$= ABC\ \{AB \to C\}$
$= ABCX\ \{C \to X\}$
$= ABCXY\ \{X \to Y\}$
$= ABCXYZ\ \{Y \to Z\}$

$AC^+ = AC$
$= ACX\ \{C \to X\}$
$= ACXY\ \{X \to Y\}$
$= ACXYZ\ \{Y \to Z\}$
$= ACXYZB\ \{Z \to B\}$

$AY^+ = AY$
$= AYZ\ \{Y \to Z\}$
$= AYZB\ \{Z \to B\}$
$= AYZBC\ \{B \to C\}$
$= AYZBCX\ \{C \to X\}$

$AZ^+ = AZ$
$= AZB\ \{Z \to B\}$
$= AZBC\ \{AB \to C\}$
$= AZBCX\ \{C \to X\}$
$= AZBCXY\ \{X \to Y\}$

Here, $AB,\ AC,\ AY\ and\ AZ$ are candidate keys.

Prime Attributes: Attribute set that belongs to any candidate key are called Prime Attributes.
So, all attributes are prime attributes.

6. Consider a relation **student***(studID, Sname, Age, Sex)* where $studID$ is the primary key. Then, how many super keys are possible for **student**?

[NAT: 1 points]

Ans : 8

Solution:
Consider a relation $R(A_1, A_2, A_3.....A_n)$, a candidate key remaining $A_2, A_3.....A_n$ any subset of attribute which combine with A_1 is a superkey.
Total Keys $= 2^{n-1}$.
Here, n= 4, So, the number of super keys are 8.

7. Which among the following is a trivial functional dependency?

[MCQ: 1 points]

- ◯ $AB \to BC$
- ◯ $AB \to CD$
- ◯ $A \to B$
- √ $AB \to B$

Solution: In general, $\alpha \to \beta$ is trivial if $\beta \subseteq \alpha$. Hence, Option 4 is the right answer.

8. Consider a relation $R(A, B, C, D, E)$ with the following functional dependencies:

$$\mathcal{F} = \{A \rightarrow B, A \rightarrow D, D \rightarrow C, AB \rightarrow C, B \rightarrow E\}$$

Choose the attribute(s) that are extraneous to any of the functional dependencies in $\mathcal{F}$.

[MSQ: 3 points]

- ◯ A
- √ B
- ◯ C
- ◯ D

Solution: $A \rightarrow D, D \rightarrow C \Rightarrow A \rightarrow C$
It follows that in the FD $AB \rightarrow C$, B is extraneous.

9. Given relation $R(A, B, C, D, E)$ and a set of functional dependencies

$$\mathcal{F} = \{A \to B, A \to D, D \to C, AB \to C, B \to E, BD \to CE\}$$

find the prime attribute(s) of R.

[MSQ: 2 points]

- √ A
- ◯ B
- ◯ C
- ◯ D
- ◯ E

Solution: The attribute that has A in its closure is only A itself. It follows that any candidate key must contain A as a component.
However, since $A^+ = \{ABCDE\}$, it follows that A is a candidate key and hence A is the only prime attribute.

10. Consider a relation $R(A, B, C, D, E)$ having the following functional dependencies:

$$\mathcal{F} = \{A \rightarrow BCD, C \rightarrow E, B \rightarrow D, C \rightarrow D, E \rightarrow B\}$$

Let $R_1(A, B, C), R_2(A, D, E)$ be a lossless decomposition of R. From among the given options, choose a functional dependency which may be removed from $\mathcal{F}$ that makes the decomposition lossy.

[MCQ: 3 points]

- ◯ $B \rightarrow D$
- √ $A \rightarrow C$
- ◯ $A \rightarrow B$
- ◯ $E \rightarrow B$

Solution: In the decomposition $R_1(A, B, C), R_2(A, D, E)$ of R, $R_1 \cap R_2 = A \neq \emptyset$ and $R_1 \cup R_2 = R$ are satisfied.
We have $R_1 \cap R_2 = A$. If A functionally determines either R_1 or R_2, then the decomposition is lossless with respect to $\mathcal{F}$.
We have $A^+/\mathcal{F} = ABCDE$. Hence A is a candidate key and the decomposition is lossless.
Let $\mathcal{F}' = \mathcal{F} \setminus \{A \rightarrow C\}$.
$A^+/\mathcal{F}' = ABD$
It follows that $R_1 \cap R_2$ does not functionally determine either R_1 or R_2. Hence the decomposition is lossy with respect to $\mathcal{F}'$.
The decomposition does not become lossy if we remove any other FD. Hence Option 1 is correct.